2023-24 Experimental Project
MOTION OF A SLINKY UNDER GRAVITY

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I. <u>Introduction</u>

The project investigates the motion of a slinky at its free fall by analysing displacement, velocity and acceleration of it in two stages - before and after the compression. Several steps were taken to improve the accuracy of the results, namely ensuring the slinky was level before falling and calculating the parallax error. The frames obtained from a video recording the slinky's free fall were used to measure the displacement of the slinky. The graph of the displacement/time relationship was plotted, followed by deriving velocity/time and acceleration/time relationship graphs, which were all analysed afterwards.

II. <u>Experimental Methods</u>

1) Setting the experiment

1. **Setting the scale for the measurement of displacement of the slinky:** To be able to measure the starting points of the slinky and subsequently monitor the displacement of its top and bottom parts, the scale consisting of two 1-metre rulers was built with a resolution of 0.001 metres. As the scale divisions were not visible from the position of the camera, thereby making it impossible to record changes in displacement, it was decided to attach a self-made scale with a resolution of 0.02 metres next to the rulers with the scale divisions (see Fig. 1). The origin of the scale was determined to be the starting point for measuring the height of the top part of the slinky.

Fig. 1

2. **Putting the slinky into equilibrium before its free fall:** Several steps were taken to ensure that the slinky was hanging freely before falling. Three ropes of identical length and cross-sectional area were attached to the top coil of the slinky so that the distances between the three nodes are equal (see Fig. 2). The other


Fig. 2

ends of the ropes were tied together forming a single loop through which an additional rope was tied to the door's metal fixture in front of the scale while making sure that the top coil of the slinky was straight. Following the steps described, the construction of slinky and ropes attached to it were brought to equilibrium and the length of 0.613 m of a stretched slinky hanging was recorded.

3. **Recording a slow motion video of the slinky's fall:**
A GoPro camera with a slow motion function was used to produce two videos of the slinky's motion – one video depicting free fall of the slinky from rest until compression and the other video depicting the motion after compression. For the first video, the camera was fixed so that its height was equal to the top part of the slinky, whereas for the second one the camera's height matched the height of the slinky's bottom platform (see Fig. 3). When the equilibrium of the slinky was achieved, the rope connecting the door metal fixture and a node supporting the slinky was cut. The video was started when the slinky was brought to equilibrium and the recording was halted when the bottom part touched the ground.

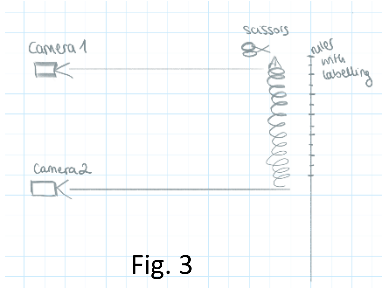


Fig. 3

2) Processing obtained videos of the slinky's free fall:

The frame rate of the camera used constituted 120 frames per second. The time between individual frames was determined as 1/120 seconds. The chosen frames that reflect the motion of the slinky were printed, and changes in displacement were recorded using the scale of 0.1 : 0.021 (0.1m is the actual size, 0.021m is the image size). As the two videos were shot, two sets of results were produced – the first set based on the first video representing the slinky's motion from rest to compression and the second based on the second video capturing the subsequent fall of the slinky starting from the state of compression. After the changes in displacement were recorded, two sets of graphs were produced respectively.

1. **Processing the first video:**
25 consecutive frames, capturing the movement of the top part of the slinky from state of rest to full compression, were selected and printed each on A4 paper. As it was discovered that the bottom part of the slinky remained stationary until full compression, changes in displacement of only the top part of the slinky were recorded and used for drawing a displacement/time graph.

2. **Processing the second video:**
   31 consecutive frames capturing the compression of slinky and its further fall were selected and printed each on A4 paper. The values of displacement of both top and lower part of the slinky after compression were recorded and used for producing a displacement/time graph consisting of two graphs representing the motion of the top and the bottom parts respectively.

3. **Processing graphs:**
   Two separate displacement/time (s/t) graphs (each depicting different stages of the slinky's free fall) were plotted in the program "Microsoft Excel", after which the line of best fit was generated and its equation was displayed by "Excel". The equation of the line of best fit was then inserted into the program "Desmos" (see Fig. 4), where the velocity values were automatically calculated by determining the gradient of the tangents (→derivative)to the previously produced line of best fit of the s/t graphs at the time intervals equalling the time between individual frames. Next, the velocity/time (v/t) graphs were plotted. The same process was used to produce acceleration/time (a/t) graphs out of the v/t graphs: finding a derivative of the functions representing a line of best fit of v/t graphs at the specific time points. For the better understanding of tendencies in the slinky's motion all the way from the rest to moving after compression, the two graphs describing the same relationship (s/t, v/t and a/t) were combined and analysed.

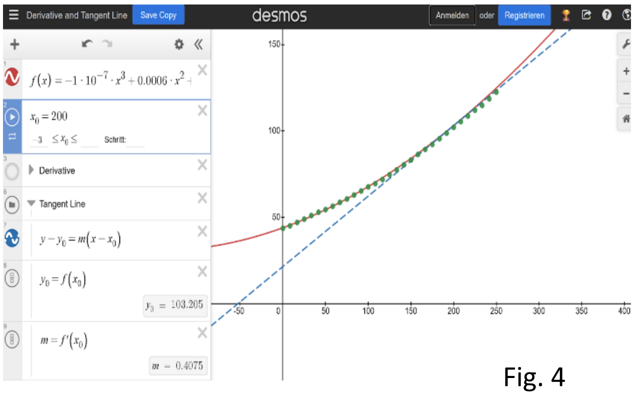


Fig. 4

III. Safety hazards and measurements

While doing this experiment several potential dangers had to be considered: a stool has been used to cut the string from which the slinky was hanging. Whilst using the stool the person cutting the cord could fall so the safety of the person had to be ensured by another person holding the stool.
Another danger were the scissors that had been used. The scissors could be a danger if someone accidentally cut themselves. If someone was to fall the scissors could have gone into their eyes so protective glasses had to be worn.

IV. Improving accuracy

In order to attain the highest accuracy, some measurements which should exclude potential uncertainty factors were taken.

1) Minimising human factors:

As a tilt of the upper slinky platform could lead to flawed assumptions about the displacement - the slinky platform surface was taken as the reference point for measurements - it was ensured that the slinky was level before falling (see Fig. 5). The same idea was applied to the second set of data collected for the subsequent motion of the slinky (2nd video capturing the movement after compression), as the change in conditions would distort the results. The levelness of the slinky was achieved with the following calculations:

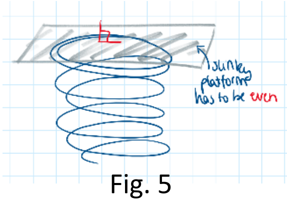


Fig. 5

The sides AB, BC, AC of the triangle ABC, shown on the diagram, represent the distances between nodes tied to the top part of the slinky, which are intended to be made equal. If to split the triangle ABC into three triangles - AOB, BOC and COA - it can be noticed that they are equal (the lateral sides are equal as they represent the radius of the circle), hence their angles are equal as well: ∠BOA = ∠BOC = ∠COA = 360° : 3 = 120° (see Fig. 6). Having the value of the radius (3.15 cm), which is the value for the equal sides of the three equal triangles and the angle between them (120°), the chords of the circle (AC, CB, AB), which represent the distance between nodes, can be found using the Cosine Law:

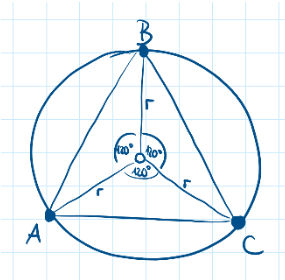


Fig. 6

$$\mathrm{AC} = \mathrm{CB} = \mathrm{AB} = \sqrt{r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos(120°)};\ \mathrm{AC} = \mathrm{CB} = \mathrm{AB} \approx 5.5\ \mathrm{cm}$$

Having calculated the distance between each two nodes attached to the top platform of the slinky, it was ensured that the distance between them was 5.5 cm.

With the intention to further minimise the human factor, one improvement was suggested: generating an electromagnetic field through electricity and using a metal slinky, and by stopping the electricity supply, the electromagnetic field would disappear causing magnetic attraction to disappear and the slinky to be released from the level position.

2) The issue of parallax:

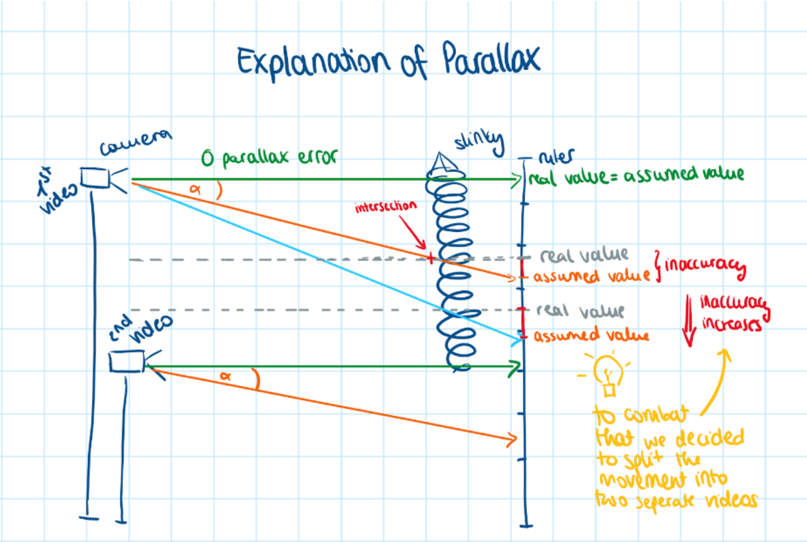


Fig. 7

The second issue that directly caught attention was the phenomenon of parallax. Parallax is the phenomenon of objects which seem to change position when viewed along different lines of sight (see Fig. 7). Relating it to this case, every picture frame, i.e. every 1/120 seconds, the height of the camera should have been adjusted to the current height of the ball to avoid Parallax inaccuracy. A camera moving at exactly the same rate as the slinky and therefore always representing the current height of the slinky would have been necessary. Due to the lack of adequate equipment, this method could not be put into practice. In the experimental setting for the first video, the camera was placed at an even level with the top of the slinky. At this height the Parallax error is zero. The further the slinky falls, the higher the deviation of the real value from the assumed value is, which leads to an increased inaccuracy in the data. Considering this, the movement of the slinky has been split into the first movement with the lower part being stationary while the top part is moving and the second movement with the entire slinky falling. These two stages of motion were captured in two separate videos. The second camera was readjusted so that it was placed at the same level with the lower part of the slinky in equilibrium (0.613m). Splitting the video was the best way to reduce the issue of Parallax, considering the technical limitations.

Parallax Calculation:

The Parallax error and the actual value of displacement were determined the following way:

The total time of 0.2 s, during which the slinky is travelling the displacement of 0.613m is known as the converted distance between the top and the lower part of the slinky; out of this information the angle changing per millisecond can be determined (see Fig. 8):

$\alpha = \tan^{-1}(0.613/1.5) = 22.2°$ (angle for a displacement of 0.613m in 0.2 s)

$\rightarrow 22.2° \triangleq 0.2$ s $\rightarrow$ $\mathbf{0.111° \triangleq 1ms}$

To determine the angle β at the correspondent time interval, the value of 0.111° has to be multiplied by the time. To calculate the inaccuracy at the given time, the tangent of the time-related angle is then multiplied by the constant factor 0.03m which represents the distance from the slinky to the door: inaccuracy i at angle β → **i = tan(β)*0.03**

The attained inaccuracy value i at the respective point has to be subtracted from the assumed value y to get the actual value a (a = y − i) as the actual value is above the assumed value; this procedure was undertaken for every single time frame of 1/120s; the inaccuracy therefore is represented by the tangents at each angle; Figure 9 illustrates the inaccuracy deviation at each angle graphically highlighting that the inaccuracy increases with the increasing displacement of the slinky. The same procedure was made with the second video, depicting the motion after the compression; however, as the height of the camera has been adjusted – it matched the height of the lower part of the slinky, the Parallax calculation changed.

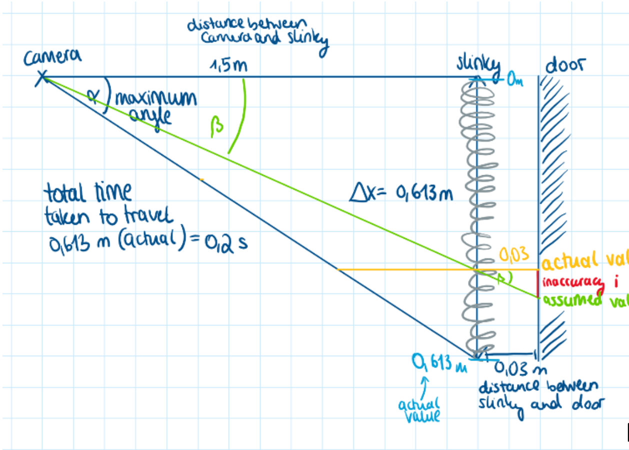


Fig. 8

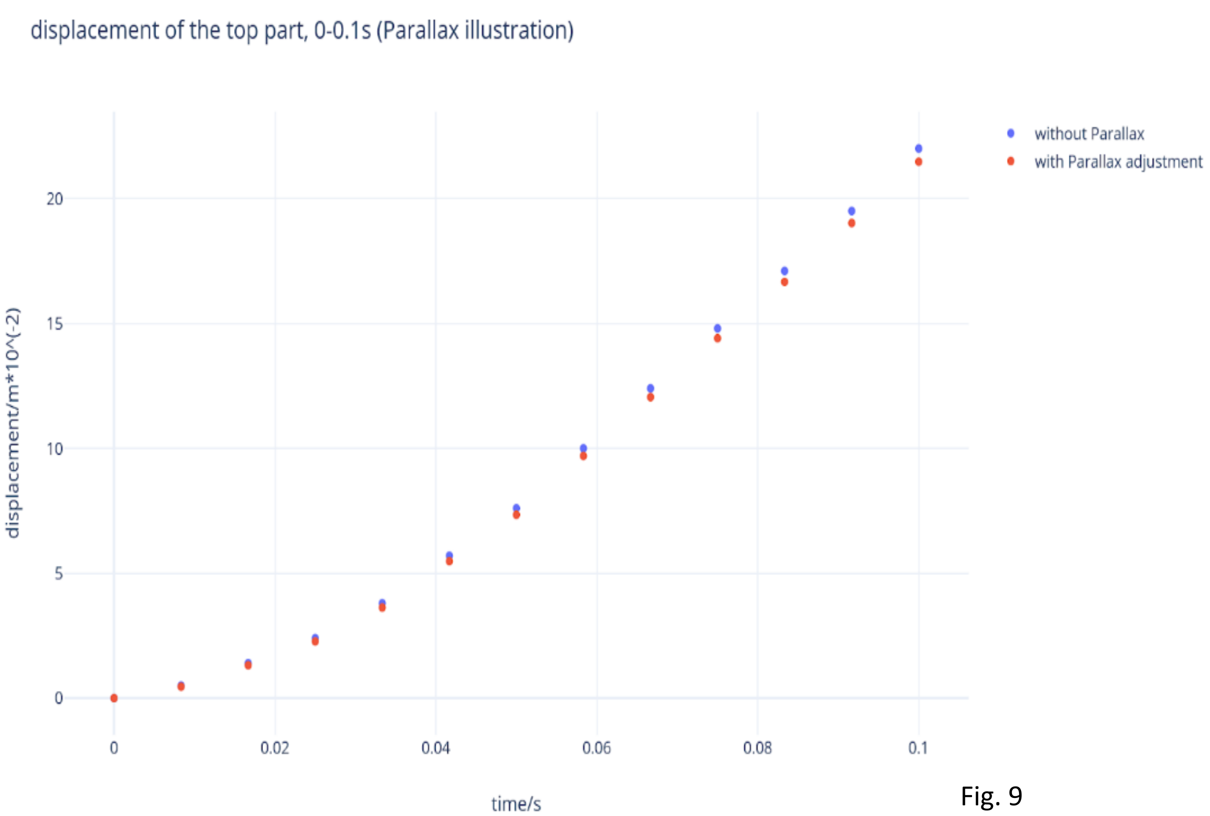


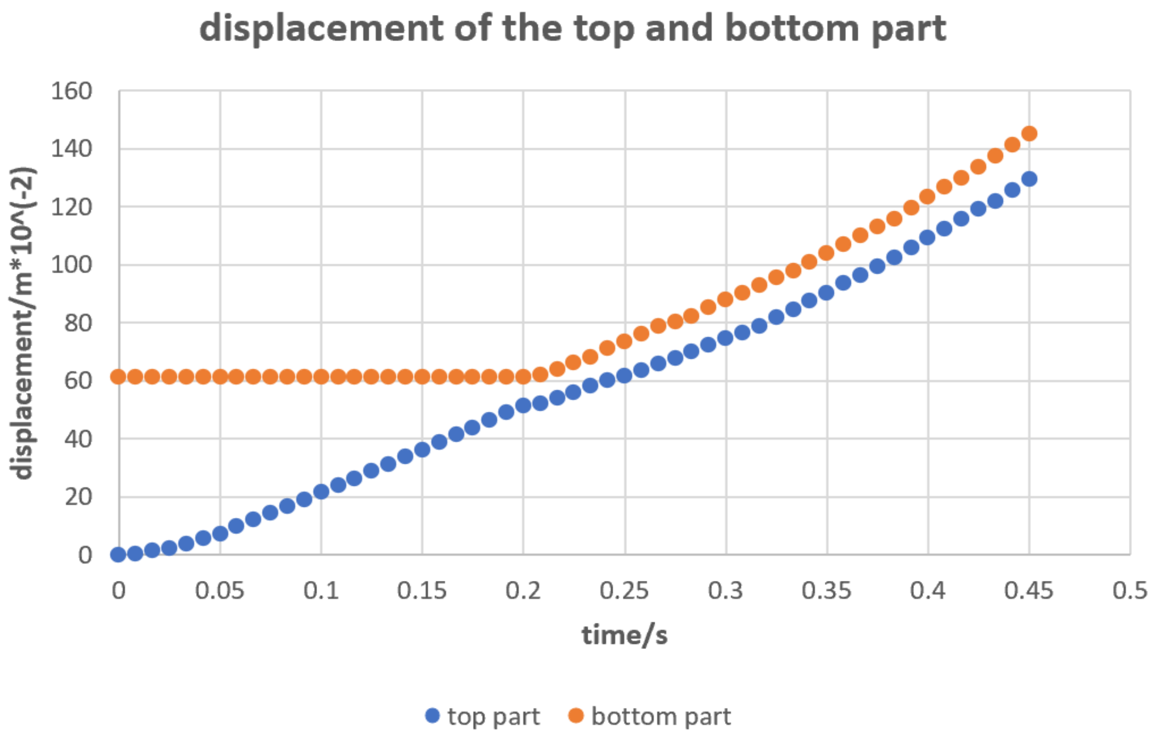
Fig. 9

3) **Using reference points:**

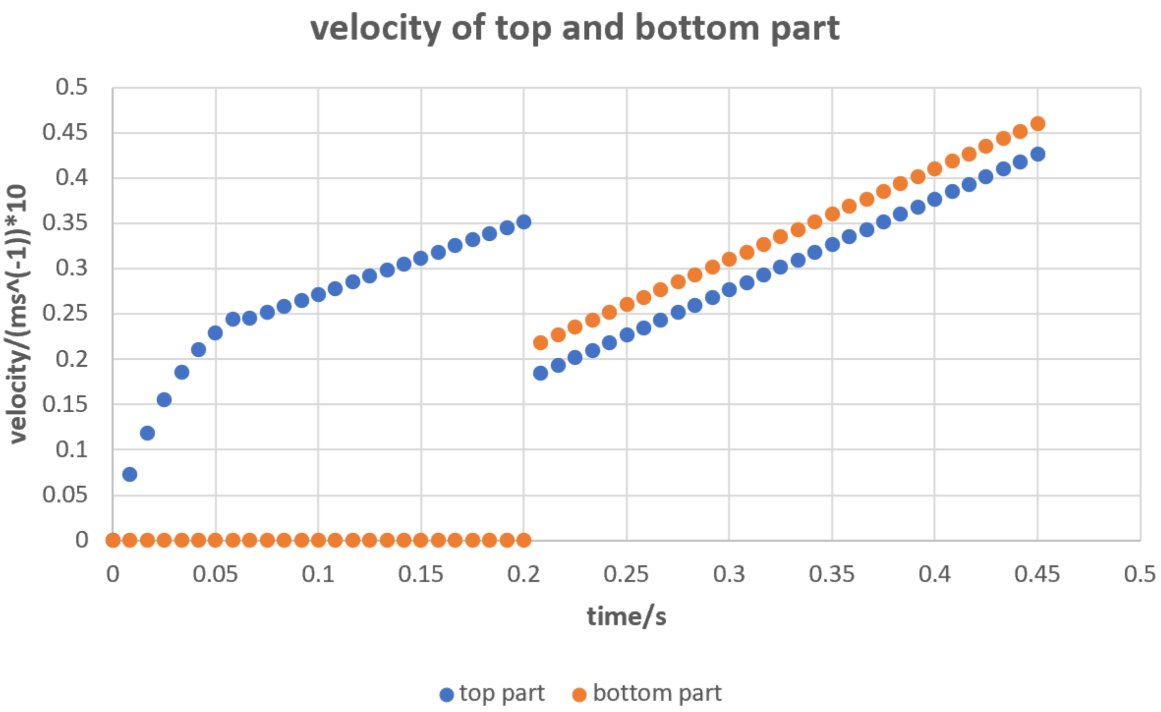
The last step taken was the employment of a colourful slinky (see Fig 10); this was helpful when identifying the exact positions of the top and bottom parts of the slinky. The colourful printings allowed a clear distinction and therefore the same points could always be considered rather than approximates of the confines of the slinky; an improvement suggestion would be to take a more accurate camera with a higher resolution so that the confines of the slinky are better visible; in addition the markings at the door would then be more concise and a conversion with a scale factor would not have been necessary.

Fig. 10

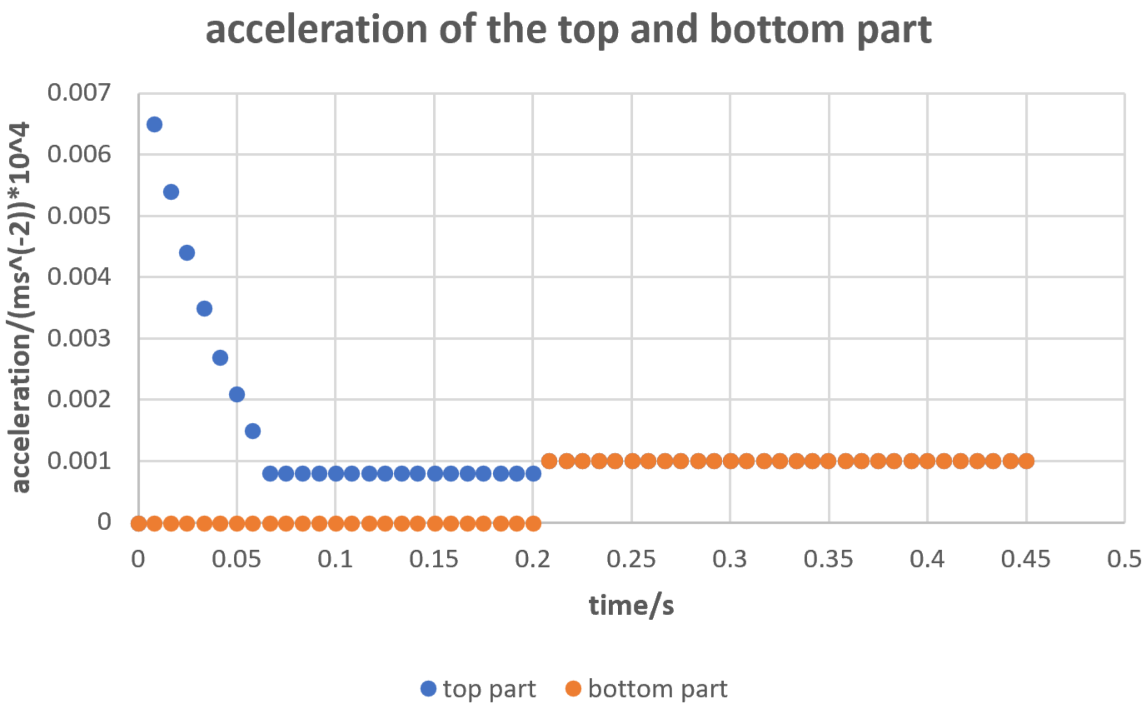
V. Analysis of the results



Time frame/ms	part	Equation of the line of best fit for S/t graphs used for finding the velocity values
0 - 58.3	top	5*10^(-8)x^4 - 2.09*10^(-5)x^(3) + 0.0034x^2 + 0.0211x - 0.0108
66.7 - 200	top	0.0004x^2 + 0.1917x - 2.0466
200 - 450	top	0.0005*x^2 + 0.0298*x + 35.939
200 - 450	bottom	0.0005*x^2 + 0.0324*x + 35.912



Time frame/ms	part	Equation of the line of best fit for v/t graphs used for finding the values of acceleration
0 - 200	bottom	y = 0
0 - 75	top	y = 3*10^(-7)x^3 - 8*10^(-5)x^2 + 0.0078x + 0.0063
83.3 - 200	top	y = 0.0008x + 0.1917
200 - 450	bottom	y = 0.001x + 0.0184
200 - 450	top	y = 0.001x - 0.0152



1) **Analysis of the top part of the slinky - before compression (0-0.2s):**
The a/t-graph shows a decreasing acceleration of the top part of the slinky for the first 0.067s. It can be seen on the v/t-graph that velocity in this time interval (until 0.067s) falls at an exponential decay. From 0.067s to 0.2s, the top part of the slinky is moving at a constant acceleration of about 8 m^(-2), which is close to the gravitational acceleration. In this period of time, the velocity is increasing linearly, reaching a maximum velocity of 35ms^(-1). The acceleration, acting on the top part of the slinky, is 6.5 times higher than the gravitational acceleration of 9.81 ms^(-2). This finding raises the question why the slinky is accelerating at a higher rate than the gravitation. The diagram of forces has been used to illustrate this phenomenon.

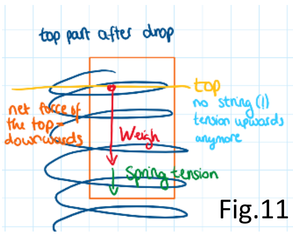


Fig.11

Initially, the slinky is stationary as the string holding the slinky is exerting a force upwards which is equal to the total force acting downwards. As the slinky is dropped, the tension causing the upward force disappears (see Fig. 11), leading to the net force acting downwards and the slinky accelerating downwards at an initial rate of 65 ms^(-2), which is 6.5 times as much as the gravitational acceleration. Thus, there must be an additional force acting downwards, which is the spring tension of the slinky, caused by the expansion of the spring from its equilibrium state. Spring force (H) is calculated by the multiplication of the spring constant (c) and the displacement of expansion (d) of the spring according to the Hooke's law (H = d*c ∴ H ∼d). As the slinky travels down, the expansion distance decreases, therefore the spring tension decreases, together with the total downward-acting force, as force F = weight (W) + spring tension (H) (see Fig. 12). According to the Newton's second law of motion (F=m*a ∴ F∼a), the total acceleration downwards must decrease if the total downward-acting force F is decreasing, which explains why the acceleration is decreasing. After 0.067s, the slinky is moving at a constant acceleration of about 8 ms^(-2) which is almost equal to the gravitational acceleration. It can be assumed that at 0.067s, the top part of the slinky is reaching the centre of mass and the spring tension acting on the top part of

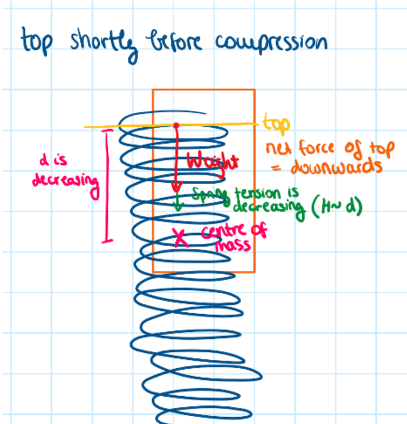


Fig.12

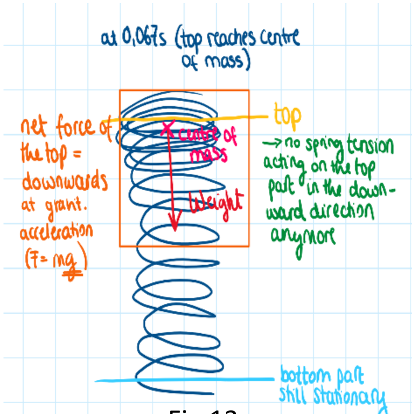


Fig.13

the slinky is zero, as the displacement d from the centre of mass is zero (if assumed the extension of the slinky is from the initial position of the top part to the centre of mass of the slinky) (see Fig. 13). So, the only force acting on the top part is the weight; the net acceleration of the top part of the slinky therefore is equal to the gravitational acceleration (F=mg), which might explain why the top of the slinky suddenly accelerates at a rate 8 ms^(-2).

2) **Analysis of the lower part of the slinky - before compression:**
It was observed that the lower part of the slinky remains stationary during the slinky's movement from rest to the full compression, which means that there is an upward force equalling weight (see Fig. 14). This force could be the tension force that disappears once the slinky is compressed, causing the slinky to accelerate only due to gravity.

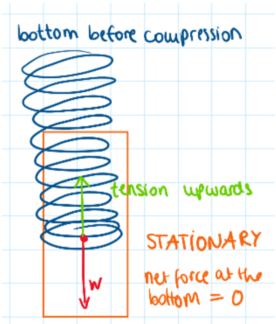


Fig.14

3) **Analysis of the top and lower part of the slinky - during/after compression**
The a/t graph suggests that slinky moves at a constant acceleration of 10ms^(-2) from 0.2s – the point of compression – to the rest of the recorded motion. After compression, the top and lower part of the slinky are moving together at the same rate – velocities of both parts increase linearly. The suggested reason for this is the absence of the spring tension when the slinky is fully compressed, therefore the net resultant force downwards is only comprised of the weight of the slinky (see Fig. 15). However, the question may arise why the bottom part of the slinky is moving at a higher velocity than the top part; it is assumed that this is due to impulse and the energy conversion principle. As compression occurs, a large part of the slinky's top part kinetic energy, gained by the increasing velocity at its peak of 35ms^(-1) at 0.2s, is transferred to the bottom part of the slinky, while another part of it is converted into other energy forms, such as thermal energy caused by the high friction force during collision. The decrease in the kinetic energy of the top part during compression explains the sharp drop of its velocity at 0.2s. As the bottom part of the slinky gains kinetic energy from the top part through the impulse, the velocity of the lower part increases drastically from state of rest, even surpassing the velocity of the top part at the same time.

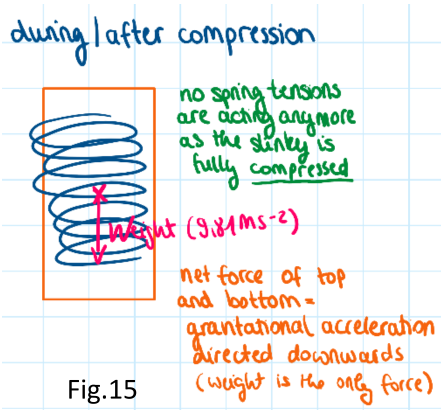


Fig.15

VI. Uncertainty consideration

The values of displacement of the slinky were measured on the printed A4 paper frames with a ruler of 0.001 metre resolution before calculating the actual values of displacement according to the proportion of 0.021 metre (the image size) : 0.1 metre (the real value). The combined uncertainty based on the identical scale division of both rulers is +- (0.0005+0.0005) = +- 0.001, which means that all the physical quantities' values measured or derived can be 0.001 units larger or smaller according to the determined uncertainty.

VII. Conclusion

The top part of the slinky first accelerates at a higher rate than the gravitational acceleration where it decreases until it almost reaches a constant acceleration of 8ms^(-2) which is close to the gravitational acceleration of 9.81 ms^(-2). Meanwhile, the lower part stays stationary until the compression of the slinky. Subsequently the top part along with the lower part of the slinky both move at the same rate with an acceleration of 10ms^(-2), which again is close to the value of gravitational acceleration of 9.81ms^(-2).

END OF REPORT